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LETTER TO THE EDITOR

Quantum fluid dynamics description of a multiterm potential nonlinear Schrödinger–Langevin equation

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Abstract. Via quantum fluid dynamics we present a general method to obtain a solution of the nonlinear Schrödinger-Langevin equation with a multiterm potential for the description of interactions in non-conservative systems. The nonlinear complete set of equations forming the basis of our model are derived. The corresponding solutions are exhibited in terms of auxiliary ordinary differential equations which provide relations between the potential coefficients and other coefficients prescribing the fluid-particle behaviour.

In recent years quantum mechanical treatment of dissipative processes has been a subject of much interest due to its applicability to solid state physics, statistical physics, photochemistry, fission and heavy ion physics (Dekker 1981, Messer 1979, Hasse 1978 and references therein, Nassar 1984). At the same time, the study of anharmonic oscillator-like interactions in conservative systems has evoked much attention because of its varied application in field theory and molecular physics (Datta and Rampal 1981, Sharma and Sharma 1984). In particular, exact solutions of the Schrödinger equation with various multiterm potentials, for the description of interactions in conservative systems, have been found (Leach 1984, 1985, Flessas and Watt 1981, Flessas 1981).

In this paper, we complete an investigation started in an earlier work (Nassar 1984), generalising the method presented there in order to obtain a solution of the nonlinear Schrödinger-Langevin equation (NLSLE), for the description of interactions in non-conservative systems (Kostin 1972, Nassar 1985), with a multiterm potential. To this end, we begin with the NLSLE, which has found use in many applications (Weiner and Forman 1974, Griffin and Kan 1976)

$$i\hbar \frac{\partial \psi}{\partial t}(x,t) = -\frac{\hbar^2}{2m}\frac{\partial^2 \psi}{\partial x^2}(x,t) + \left(\frac{\hbar \nu}{2i}\ln\frac{\psi(x,t)}{\psi^*(x,t)} + V(x,t)\right)\psi(x,t), \tag{1}$$

where $\psi(x,t)$ is the wavefunction, $(\hbar\nu/2i)\ln(\psi/\psi^*)$ accounts for the dissipation and the external potential is given by

$$V(x,t)/m = \tfrac{1}{2}a(t)x^2 + \tfrac{1}{3}b(t)x^3 + \tfrac{1}{4}c(t)x^4 + \tfrac{1}{5}d(t)x^5 + \tfrac{1}{6}e(t)x^6 - f(t)x. \tag{2}$$

To obtain a fluid dynamical description of the wavefunction $\psi(x,t)$, we express this function in the Madelung polar form

$$\psi(x,t) = \phi(x,t)\exp(iS(x,t)). \tag{3}$$

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After substitution of (3) into (1) we obtain from its real and imaginary parts

$$\partial v/\partial t + v\,\partial v/\partial x + \nu v = -(1/m)(\partial/\partial x)(V + V_{qu}) \tag{4}$$

and

$$\partial \rho/\partial t + (\partial/\partial x)(\rho v) = 0 \tag{5}$$

where $\rho = \phi^2$ is the quantum fluid density, $v = (\hbar/m)\,\partial S/\partial x$ is the fluid submicroscopic velocity, and $V_{qu} = -(\hbar^2/2m)(1/\sqrt{\rho})\,\partial^2\sqrt{\rho}/\partial x^2$ is the Bohm quantum potential. An essential feature of the quantum potential is that the force arising from it is not like an ordinary mechanical force. Rather, it acts more like a self-active information content. So it follows that the expectation value of the Bohm quantum force vanishes for all times, i.e., $\langle \partial V_{qu}/\partial x\rangle = 0$. $\langle F\rangle \equiv \int F\rho\,\mathrm{d}x$ is the expectation value of F taken over an ensemble of equivalent particles.

This suggests that (4) can be split into (Nassar 1984)

$$\partial v/\partial t + v\,\partial v/\partial x + \nu v + ax + bx^2 + cx^3 + dx^4 + ex^5 - f = W(x - X, t) \tag{6}$$

and

$$\frac{\partial}{\partial x}\left(\frac{\hbar^2}{2m^2}\frac{1}{\sqrt{\rho}}\frac{\partial^2\sqrt{\rho}}{\partial x^2}\right) = W(x - X, t) \tag{7}$$

provided that $\langle W(x - X, t)\rangle = 0$.

Supposing the ansatz $\rho(x, t) = (N(t))^{-1/2}\exp[-\alpha(x - X)^2 - \frac{1}{2}\beta(x - X)^4]$ we may write (7) as

$$\frac{\partial}{\partial x}\left(\frac{\hbar^2}{2m^2}\frac{1}{\sqrt{\rho}}\frac{\partial^2\sqrt{\rho}}{\partial x^2}\right) = A(t)(x - X) + B(t)(x - X)^3 + C(t)(x - X)^5, \tag{8}$$

where $A(t) = (\hbar^2/m^2)(\alpha^2 - 3\beta)$, $B(t) = (4\hbar^2/m^2)\alpha\beta$ and $C(t) = (3\hbar^2/m^2)\beta^2$. For simplicity, let $\beta = \alpha^2$. Then

$$W(x - X, t) = -p(t)(x - X) + q(t)(x - X)^3 + r(t)(x - X)^5, \tag{9}$$

where

$$p(t) = 2\hbar^2\alpha^2/m^2, \qquad q(t) = 4\hbar^2\alpha^3/m^2 \qquad \text{and} \qquad r(t) = 3\hbar^2\alpha^4/m^2. \tag{9a, 9b, 9c}$$

Now, by requiring the condition of normalisation $\int_{-\infty}^{+\infty}\rho\,\mathrm{d}x = 1$ we have

$$\int_{-\infty}^{+\infty}\exp[-\alpha(x - X)^2 - \tfrac{1}{2}\alpha^2(x - X)^4]\,\mathrm{d}x = (N(t))^{1/2} \tag{10a}$$

or

$$\int_{-\infty}^{+\infty}\exp[-\tfrac{1}{2}\alpha^2 z^2 - \alpha z]z^{-1/2}\,\mathrm{d}z = (N(t))^{1/2} \tag{10b}$$

with $z = (x - X)^2$.

Since (Gradshteyn and Ryzhik 1965)

$$\int_0^{\infty}\exp(-bz^2 - az)z^{m-1}\,\mathrm{d}z = (2b)^{-m/2}\Gamma(m)\exp(a^2/8b)D_{-m}(a/\sqrt{2b}) \tag{11}$$

we find

$$N(t)=\pi k^2\xi(t), \tag{12}$$

where $k\equiv e^{1/4}D_{-1/2}(1)$ and $\xi(t)\equiv 1/\alpha(t)$. Notice that ξ has dimensions of $[x]^2$. So

$$\rho(x,t)=\frac{1}{(\pi k^2\xi(t))^{1/2}}\exp\left(-\frac{(x-X)^2}{\xi(t)}-\frac{(x-X)^4}{2\xi^2(t)}\right). \tag{13}$$

Substituting (13) into (5) and integrating we get

$$v(x,t)=(\dot{\xi}/2\xi)(x-X)+\dot{X}, \tag{14}$$

where the constant of integration must be zero since ρ vanishes for $|x|\to\infty$.

Next, by replacing (14) and (9) into (6) and after some easy manipulations we are left with

$$\begin{aligned}(\ddot{\xi}/2\xi-\dot{\xi}^2/4\xi^2+\nu\dot{\xi}/2\xi+a+p-3qX^2-5rX^4)(x-X)+x^5(e-r)+x^4(d+5rX)\\ +x^3(c-q-10rX^2)+x^2(b+3qX-10rX^3)\\ +(\ddot{X}+\nu\dot{X}+aX-2qX^3-4rX^5-f)=0.\end{aligned} \tag{15}$$

This equation is identically satisfied if

$$\ddot{\xi}/2\xi-\dot{\xi}^2/4\xi^2+\nu\dot{\xi}/2\xi+a+2\hbar^2/m^2\xi^2-12\hbar^2X^2/m^2\xi^3-15\hbar^2X^4/m^2\xi^4=0 \tag{16a}$$

$$\ddot{X}+\nu\dot{X}+aX-2qX^3-4rX^5=f \tag{16b}$$

$$b+3qX-10rX^3=0 \tag{16c}$$

$$c-q-10rX^2=0 \tag{16d}$$

$$d+5rX=0 \tag{16e}$$

$$e-r=0, \tag{16f}$$

where we have used ($9a, 9b, 9c$) in ($16a$).

To summarise, we have presented a solution to a NLSLE (describing interactions in non-conservative systems) with a multiterm potential in terms of auxiliary differential equations. We believe that the method carried out above poses some perspectives and an alternative stepping stone for applications and further work both from the analytical and numerical point of view.

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